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UNITED STATES DEPARTMENT OF AGRICULTURE
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H. H. BENNETT, CHIEF

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THE TRANSPORTATION OF SEDIMENT BY FLOWING WATER

By

J. W. Johnson

Presented at a Colloquium
Department of Civil Engineering
Columbia University
New York, N. Y.
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Sedimentation Division
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Adequate knowledge of the principles of transportation and deposition of erosional debris is essential to such varied engineering problems as river regulation and soil conservation, and to those encountered in the construction of harbors, locks, irrigation and ship canals, water and sewage-treatment plants, and hydroelectric plants. Perhaps the important objective of many sediment-load studies is the determination of potential or actual damage to reservoirs, stream channels, drainage ditches, and canals.

At the present time two methods are used for predicting the life expectancy of a reservoir, (1) by use of silting rates estimated from sediment-load measurements, and (2) by analogy with measured rates of silting in nearby existing reservoirs.

For application of the latter method a considerable volume of basic data have been collected by the Soil Conservation Service (9, 1)[3] and other agencies (53).

To the engineer, the character and amount of sediment in a particular channel are of primary interest. To make the picture more complete, however, a brief discussion of the source of sediment and the manner in which it reaches a stream channel is also of interest.

From the standpoint of hydraulics, a drainage basin should be regarded as a combination of two distinct erosion zones, each governed by a special set of principles and factors. These are (1) a zone of concentrated flow consisting of actual drainage channels, streams, and flood plains, in which stream erosion is the prime factor, and (2) a zone of unconcentrated flow, constituting the remainder of the drainage basin, in which rain wash and sheet erosion

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[1]This paper is intended primarily for the purpose of giving a broad, general description of the problem of sediment transportation; however, ample references are given for the benefit of those who wish to study in detail the various phases of the problems.

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[3]Numbers in parentheses refer to Literature Cited p. 20.

are the prime factors. For the first zone there is urgent need of information on the principles of erosion, entrainment, and transportation of debris in definite channels; and for the second zone there is need for evaluation of the relationship of flow to land use practices, topography, shape, and size of drainage area, vegetative cover, surface and underground storage, and climatic conditions.

Since 1918 numerous studies of the relation of erosion to slope, land use, amount and intensity of rainfall, etc., have been conducted on small plots. More recently somewhat similar investigations have been initiated on small watersheds at a number of localities by the Soil Conservation Service. The discussion to follow will be restricted, however, to the movement of sediment in defined channels.

In studying the problem of sediment transportation in such channels consideration must be given separately to the two common types of movement, as the laws governing them differ materially. These two types of movement are suspension and traction. The material carried in suspension consists usually of very small particles carried in the water free from contact with the bed for considerable periods of time. Material transported by traction is commonly known as bed load, since it is rolled or pushed along the bed of the stream.

The method of studying these two modes of transportation differs in that bed-load investigations have been limited mostly to the laboratory, while suspended-load investigations have been limited mostly to the field. Such procedure has been established mainly for reasons of economy, as the cost of apparatus for making bed-load measurements in the field is almost prohibitive as compared to the expense of laboratory studies, and vice versa for suspended-load studies. It is of interest to note that although the bed-load laws are difficult and expensive to determine, once determined they will have general application to any alluvial stream.

On the other hand, the total suspended load, although easily and inexpensively measured, is more difficult, if not impossible, to relate to any definitely and easily observed hydraulic factors of a stream. This latter condition is due to the variability in quantity of the fine material which, in most cases, constitutes the predominant portion of the suspended load. Such fine material must originate in the zone of sheet wash or in the tributaries, as it is generally not available in the bed of the stream for many miles above the point of observation. This factor explains the general lack of any fixed relationship between the discharge and the load of finer material. In other words, the relatively coarse material moved as bed load in a stream is abundantly available in the bed, and its movement is related to the stream's energy, and

consequently to discharge; whereas, the finer material which moves as suspended load must originate on the areas of sheet erosion and its supply is therefore controlled by variable conditions within the drainage basin. A stream usually can carry a suspended load many times greater than that observed by sampling, but in very few cases is sediment delivered by tributaries and rain wash in sufficient quantities to fully "saturate" a stream.

Primarily, the purpose of sediment-load investigations is to obtain basic data relating to the magnitude and nature of the sedimentary load and to determine the fundamental laws governing the sedimentary characteristics of streams. It should be noted that although two streams may be similar to their hydraulic characteristics; namely, slope, rate of discharge, etc., they may differ greatly in their sedimentary characteristics, magnitude of sedimentary load, and mechanical composition of the sediment.

Investigations are being made to determine the laws governing sediment transportation and to verify in nature one or several of the existing formulas that have been developed in laboratory experiments. The applicability of laboratory experiments to a river depends, to a great extent, on the degree of hydraulic similarity that is attained between a laboratory channel and a natural stream. Obviously, exact similarity cannot be attained, but if the laboratory experiments are carried on over a wide range of the conditions governing the similarity, the transferability of results to natural streams will be more successful. Hence, the measurements made in a river should reflect and be guided by the experience gained in the laboratory.

It is at once apparent that there are a number of serious difficulties inherent in the use of a natural stream as a means of studying the effect of the several variables. For instance, the bed load passing any section varies to a greater or lesser extent with the discharge, slope, cross-sectional area or hydraulic radius, which in turn affect the mean velocity, hydraulic roughness, the availability, characteristics of sediment, and possibly others. Measurements of bed-load transportation in a river will indicate the net effect of all the variables as they interact upon each other, and not the separate effect of each. The manner and degree in which each variable affects the movement of bed load is a subject for laboratory experiment to determine how other variables can be controlled. An equation developed through laboratory experiments and verified by actual river measurements then can be used with reasonable assurance.

BED-LOAD TRANSPORTATION

The suspended load of sediment in streams can be measured by taking samples at a sufficient number of points in a cross section, but the problem of ascertaining the extent of movement of bed load has not yet been solved. Except for a few experiments conducted in natural streams with specially constructed traps, quantitative studies of transportation of bed load have been limited almost entirely to laboratory flume studies. In no case does it appear that former work has included actual quantitative measurements of sediment in transit, in the form of bed load, in such streams as are large enough to be generally representative of rivers on which such knowledge is desired. Although many serious and admirable efforts have been made to develop rational formulas for expressing tractive power of streams and to apply the data from flume studies to natural streams, the various methods suggested do not fully and adequately permit the determination of actual volumes of material transported in such streams.

Laboratory Investigations

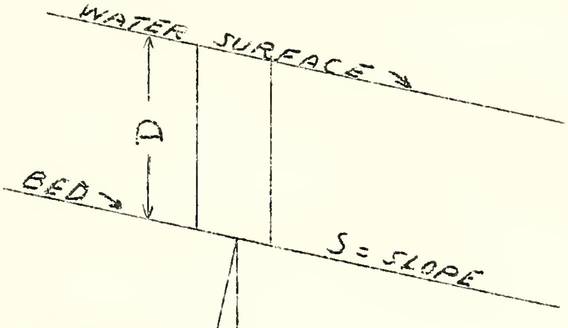
Since the classic experiments by Gilbert (14) in 1914, a large number of investigations on bed-load transportation in flumes have been made at various laboratories throughout the world. Particular emphasis has been given to the study of (1) the critical conditions controlling the commencement of the movement of bed materials of various sizes, and (2) the rates of their movement. A few other related problems which have been studied are: Relation between the bed-roughness coefficient and particle size and other peculiarities of the bed, non-silting channels in erodible materials, scour in river beds, sorting of sediments, and riffle formations.

The laboratory apparatus usually used has consisted of a circulating water-supply system, constant head tank, weir box, stilling chamber, tilting flume (1 to 3 feet wide, from 30 to 50 feet long), tailgate, and a return channel.

The usual experimental procedure consists of first placing sand in the flume to a depth of about 2 inches and molding the bed to a certain slope. Water is then allowed to pass through the flume under flow conditions made uniform by manipulation of the tailgate. At low discharges no movement of material occurs, but at a certain discharge sediment movement begins and increases in rate as the discharge increases. A trap is provided at the downstream end of the flume to measure the rate of movement of material under test. In order to insure that no progressive change occurs in the elevation or slope of the bed during the course of a test, sand is introduced at the upper end of the flume to balance the amount that is moving

and hence is caught in the trap. Observations are made, during a test, on the hydraulic and sedimentary characteristics at various discharges above the critical discharge.

<u>Critical conditions for movement</u>. The most common conception of the mechanics of bed-load movement is that a dragging force is exerted on the bed of a stream by the flowing water. This force, termed the tractive force, is the dragging or entraining force T exerted at the base of a prism of water of unit area of the bed and height D sliding, under the influence of gravity, down an inclined plane having a slope S. This may be more specifically expressed as the unit shearing force in a wide channel of depth D at zero distance from the bottom, that is,



$$T = \gamma' D S$$

where γ' = weight per unit volume of water.

Critical tractive force is that tractive force which creates "general movement" of the bed material, and is designated by the term T_c. General movement, as used in this sense, is that condition under which sand grains up to and including the largest size available are in motion.

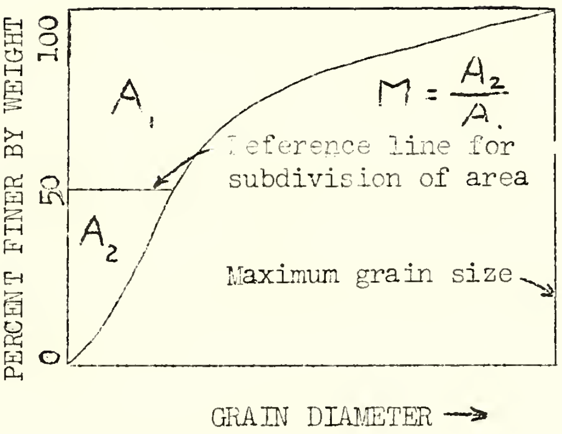
Considering the critical tractive force as related to the mechanical properties of the bed material, it can be stated that T_c is a function of grain size, grain distribution, grain form, and unit weight in water.

Flume experiments have been conducted to obtain the relation between these various factors at the point of general movement. Kramer (22), one of the first to attempt to formulate a criterion for defining critical conditions, used experimental data from all available sources and developed the following practical formula in terms of sand size and distribution:

$$T_c = \frac{100}{6} \frac{d}{M} \left(\gamma_s' - \gamma' \right) \qquad (1)$$

where T_c is the critical tractive force in grams per

square meter; d the mean grain diameter in millimeters; $(\gamma_1 - \gamma)$ the unit weight of the material in water in grams per cubic centimeter; and M is the uniformity modulus which is derived from the gradation curve of the bed material (see sketch). In later investigations at the United States Waterways Experiment Station (54) Kramer's criteria and data were used and the following modified formula was derived:



$$T_c = 29\sqrt{\frac{d}{M}(\gamma_1 - \gamma)} \qquad (2)$$

All terms are expressed in the same units as in equation 1. More recently Chang (4) analyzed all available data and also presented a formula for the critical tractive force in terms of the mechanical properties of the material. Numerous experiments conducted at the University of Iowa (32) resulted in the establishment of a relationship between a competent bottom velocity, corresponding to impending motion of the stream bed, and the size and specific gravity of the bed material; that is

$$V_c = \frac{1}{2} d^{\frac{4}{9}}(s-1)^{\frac{1}{2}}$$

where V_c = bottom velocity in feet per second; d = diameter in millimeters, and s = specific gravity of the bed material.

Perhaps the most sound approach to the problem is that of Shields' (45)[4] in which the relation of the effective tractive force to the resistance of a layer of granular material is a universal function of the ratio of grain size to the thickness of the laminar boundary layer. The resistance of a grain (that is, the force required to dislodge a grain from a bed of uniform grain size) is proportional to the weight of the grain; thus

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[4]A summary of Shields' paper is given by Hunter Rouse in An Analysis of Sediment Transportation in the Light of Fluid Turbulence. U. S. Soil Conser. Serv. TP-25, July 1939. (Mimeographed.)

$$K_1 = k_1(\gamma_1 - \gamma)d^3 \tag{3}$$

where k_1 is dependent on porosity of the uniform bed and the grain shape and γ_1 and γ are the specific weight of the grain and fluid, respectively. The effective force of the flow upon the grain is

$$K_2 = k_3 \zeta d^2 \gamma \frac{U_c^2}{2g} \tag{4}$$

where k_3 depends upon the shape of the grain of diameter d ; $\gamma U_c^2/2g$ indicates the stagnation pressure of a critical velocity; and ζ represents the resistance coefficient of the grain at a velocity U_c and is proportional to $U_c d/\nu$.

Equation 4 is reduced by applying the following law of Nikuradse for flow in rough pipes.

$$\frac{U}{V^*} - 5.75 \log \frac{y}{d} = f_1\left(\log \frac{V^* d}{\nu}\right) \tag{5}$$

where
U = local velocity
V^* = $\sqrt{gRS}$ = friction velocity
g = acceleration of gravity
R = hydraulic radius
S = slope of energy gradient
y = distance from wall
d = mean grain diameter
ν = kinematic viscosity

Equation 5 also may be written

$$U_c = f_2 V^*\left(\frac{V^* d}{\nu}\right) \tag{6}$$

in which case

$$\zeta = f_3\left(\frac{V^* d}{\nu}\right) \tag{7}$$

whereupon equation 4 becomes

$$K_2 = k_4 d^2 \gamma R S\left(\frac{V^* d}{\nu}\right) \tag{8}$$

The force of the flow exerted upon the grain at the beginning of movement becomes equal to the force required to start a grain moving; thus equating 3 and 8

$$\frac{\gamma RS}{(\gamma_1 - \gamma)d} = \frac{T_c}{(\gamma_1 - \gamma)d} = f_5\left(\frac{V^* d}{\nu}\right) = f_6\left(\frac{d}{\delta}\right) \tag{9}$$

where now

T_c = critical tractive force
$V^* d/\nu$ = Reynolds number of grain
f_5, f_6 = functions dependent upon grain shape
δ = $C\frac{\nu}{V^*}$ boundary layer thickness [see Von Karman (20)].

Shields conducted a series of experiments on the initial movement of uniform materials having a wide range in specific weight and plotted his results according to equation 9 (see fig. 4 of Rouse).[5] Such plotting emphasized the great similarity of the functional trend to that of measurements in rough pipes. In other words, the initial movement of bed particles is directly influenced by the conditions of motion in the neighborhood of the bed. Part of the plotted curve, a 45° line, representing undisturbed laminar flow at the bed, follows the equation

$$\frac{T_c}{(\gamma_1 - \gamma)d} = \frac{0.01}{\frac{d}{\delta}} \tag{10}$$

Similarly, a horizontal line representing fully developed turbulence of the bed follows the equation

$$\frac{T_c}{(\gamma_1 - \gamma)d} = 0.06 \tag{11}$$

Obviously, initial movement may occur within the laminar boundary layer, but the influence of viscosity upon such movement

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[5]See footnote 4, p. 6.

steadily decreases as the boundary layer is broken up by grains of increasing relative size. The study of Shields is of importance because it shows the effect of viscous action on the beginning of movement.

Rate of bed-load movement. The results of flume studies have been formulated to show the rate of transportation in terms of variables which the various investigators considered to best fit their data.

Most investigators have used a form of DuBoys' equation (8) wherein rate of transportation is a function of tractive force, that is

$$G = \Psi T(T - T_c) \tag{12}$$

in which G is the rate of sand movement in weight per unit time, T is the unit tractive force on the bed, T_c is the critical tractive force required to start general movement, and Ψ is a function of the size, gradation, etc., of the sand grains. $(T - T_c)$ is often referred to as the excess tractive force. Although the DuBoys equation was founded on the since disproved assumption that sand is moved in layers, it apparently describes bed-load movement as well as any other available equation.

Figure 1 shows a linear plot of rate of sand movement against tractive force for data collected at the United States Waterways Experiment Station, Vicksburg, Miss. (54). Figure 2 shows a logarithmic plot of the same experimental data according to equation 12. It is of interest to note that figure 1 permits a graphical determination of the critical tractive force as T_c is the value of T when $G = 0$. In most experiments the graphical value of T_c agrees very closely with the value calculated from observed values of depth and slope when general movement is visually noted.

Some investigators have considered slope, discharge, and grain size as being of primary importance, an example of which is the formula of MacDougall (31)

$$G = aS^b(Q - Q_c) \tag{13}$$

in which "constants" a and b depend on grain size and gradation, Q is the discharge, and Q_c is the critical discharge or that discharge at which general movement begins.

For application to tidal channels and similar environments where the ordinary hydraulic functions (such as slope) used in calculating tractive force in streams have no significance, the rate of bed-load movement has been related to bottom velocity (32, 33); that is,

$$G = c\,(V_b - V_c)^m \tag{14}$$

where $(V_b - V_c)$ represents the excess bottom velocity. Bed-load transportation has also been related to the mean velocity of a stream (37),

$$G = \text{function}\left(\frac{V_m}{R^{\frac{1}{3}}}\right) \tag{15}$$

where V_m = velocity, and R = hydraulic radius.

Many of the various flume experiments have been performed with granular material of uniform grain size. Most of Gilbert's experiments were of this nature. Later Schoklitsch (43), using some experimental data of his own in addition to that of Gilbert's, developed the following formula for uniform quartz grains:

$$G = \frac{86.7}{\sqrt{d}}\,S^{1.5}\,(Q - Q_c) \tag{16}$$

in which G is the bed load in pounds per seconds, d the grain diameter in inches, S the slope, and $(Q - Q_c)$ the excess discharge in cubic feet per second. The value of Q_c is:

$$Q_c = \frac{B(0.00532\ d)}{S^{\frac{4}{3}}} \tag{17}$$

where B is the channel width in feet.

Shulits (46) has attempted to apply this formula for uniform grain material to mixtures. The mixture is presumed to be made up of a number of grades of mean grain diameters, d_a, d_b, d_c, etc., the division into grades being quite arbitrary, and the percentage weight (a, b, c, etc.) of each grade being determined from the mechanical composition curve. The weight of bed load (G_a, G_b, G_c, ...) is then computed for each diameter d_a, d_b, d_c, ...) for the given discharge and slope with the aid of equation 16. The weight of bed load for the mixture then is:

$$G = a\,G_a + b\,G_b + c\,G_c \;---\tag{18}$$

It may happen that Q_c of one of the coarse grades is larger than Q , the instantaneous discharge. This will result in a negative bed load of the particular grade, which is substituted in equation 18 with the negative sign. It should be noted, however, that this application of the Schoklitsch formula to mixtures has been seriously questioned by many authorities.

Another formula for uniform grain material is that of Meyer-Peter (34) who used Gilbert's data, as well as data collected at

Zurich, Switzerland, and developed a relationship intended primarily for obtaining similarity in the design of movable-bed models. This formula, founded on data covering a large range in slope and in grain diameter, is

$$G = \left(aSQ^{\frac{2}{3}} - bd\right)^{\frac{3}{2}} \tag{19}$$

and the equivalent critical discharge is (when G = 0):

$$Q_c = k\left(\frac{d}{S}\right)^{\frac{3}{2}} \tag{20}$$

where Q in the above formulas is expressed in weight per unit time.

In passing it is of interest to note that a considerable scattering of points is characteristic in plottings of bed-load movement. This is mainly due to the complex riffle formation by which most movement takes place. Certain experimental errors are also responsible for the poor results of many of the various studies. Most important of these errors are (1) the use of water-surface slope instead of the energy gradient, (2) the neglect of side-wall friction, and (3) the use of relatively short collection periods when observing rates of bed-load movement.

Related studies. Although the majority of bed-load experiments have been concerned with the establishment of a criterion for the beginning of bed movement and a law for bed-load transportation, numerous other studies on related problems have been made with other objectives in mind. For example, efforts have been made to arrive at a relation between the roughness coefficient, C or n , and of particle size and other peculiarities of the bed (4, 37, 12, 13, 50, 23, 38). The design of non-silting channels in erodible materials has long been a problem of great interest in irrigation and reclamation work (51, 25, 47, 16). The phenomenon of localized scour has been subjected to surprisingly little systematic study. According to some investigators, localized scour depends essentially upon the same hydraulic and sedimentary characteristics as are found in other problems of sediment transportation.

Other studies related to bed-load movement involve the sorting of river sediments (4, 49), beach erosion (55), model experiments in which bed-load transportation takes place (3, 57, 15, 48), scour in sandy river beds (59, 18, 24), and riffle formations and bed configurations (22, 4, 45, 19, 44).

Field Investigations

As previously mentioned, quantitative studies of transportation of bed load have been limited almost entirely to laboratory flume studies. A few experiments, however, have been made in

natural streams either by using specially constructed traps or by the application of data from flume studies. As early as 1898 a specially constructed trap was used by the Nicaragua Canal Commission on streams in Nicaragua in connection with the investigation of a proposed inter-oceanic canal (6, p. 374). In 1927, after a period of 34 years of observation on the Rhine, Wittmann (58) concluded that the general Schoklitsch formula (43) was applicable to that river. More recently, numerous other investigations have been conducted with various specially constructed devices, differing in detail, consisting generally of a box or pan which is allowed to settle firmly on the stream bottom for the purpose of collecting material for a limited time (52, 10, 35, 26, 2, 39). It is recognized, of course, that all such traps act as an obstruction on the stream bottom and have a tendency to cause local scouring and to divert material from the upstream end of the trap. To obtain measurements with a reasonable degree of accuracy it is necessary that the efficiency of the trap be determined by calibration in a laboratory. Perhaps the most comprehensive calibration tests (11) of this type were conducted on the Nesper traps (35) which were used on the Rhine at Zurich, Switzerland (fig. 3). The Arnhem trap recently used by Schaank (41) in Holland was designed to eliminate the necessity of expensive and possibly inaccurate efficiency determinations (fig. 4). This trap was constructed and adjusted so that its presence would not alter the flow pattern of the stream and, as a consequence, the trapping efficiency approached 100 percent. Another type of bed-load trap is a catch box in the stream bed into which it is presumed the bed load will settle and can be measured from time to time. This practice has proved unsuccessful inasmuch as a box catches the entire bed load only at extremely low stages. At other stages the currents through the box vent the materials to such an extent that the contents of the box after floods are of no significance whatever with respect to the full volumetric rate of bed-load movement.

Figure 5 shows the distribution across a section in the Rhine as measured with the Arnhem trap of Schaank (41). The load defined as bed load in this illustration is all the material moving below a height of 5 centimeters from the bottom as this is the height of the trap. Other curves show the suspended load at various distances above the average elevation of the bed. It is obvious from this illustration that numerous measurements across the width of a stream are required to accurately estimate the quantity of bed load moved past a particular section. In all such measurements, relatively long collection periods are required to average the fluctuations which are inherent in bed-load movements. This is evident, for example, from figure 6 which shows the rate of movement in the Rhine over a period of several hours.

In formulating a program for the Soil Conservation Service, recognition was given to the urgent need for extensive information on the factors affecting the amounts, occurrence, and movement of sediment (suspended and bed) in natural streams. Accordingly a comprehensive program of research was planned and construction started on a sediment-load laboratory at the Enoree River near Greenville, S. C., in the Piedmont Plateau region, for the purpose of studying the relations between sediment load and the hydraulic and physical factors of a stream (7). At a section near the center of a straight undisturbed reach of the river, facilities have been made available for observing velocity and suspended load. A systematic bed-sampling program is conducted for the purpose of evaluating changes in bed composition. A short distance downstream from the lower end of the reach, a new type of control has been constructed to permit accurate measurements of the total sediment load of the river. In the operation of this control two sampling procedures are used to determine the sediment load. One portion of the load, consisting of the relatively coarse fractions moving on or near the bed, is removed hydraulically, by pipe connection and pump, from openings in the stream bed. This material is discharged into a settling tank from which it is later removed, weighed, and analyzed for particle size. The remaining portion of the stream load, consisting of the material which does not pass into the bottom openings, is determined by suspended-load samples taken simultaneously with the pumped samples at a point immediately downstream from the bed openings. This material is also subjected to an analysis for particle-size determinations. The results of these two procedures are combined to give the total sediment load passing the control. Complete records are kept of the composition of this load with respect to the various size fractions. The suspended-load samples collected in the natural reach make possible the application and verification of the various theories of silt transportation. To supplement the river observations on sediment-load movement, a concrete flume 5 feet wide, 30 inches deep, and 50 feet long, provided with a measuring weir, a sand-feed elevator, a sand trap, and a tailgate has been installed for studying the transportation of various sand mixtures under definitely controlled hydraulic conditions. A well-equipped laboratory has been constructed for the complete analysis of all sediment samples.

SUSPENDED LOAD

Theoretical and Laboratory Investigations

In a given cross section of a flowing stream the percentage of suspended matter varies with the distance above the bed and with velocity, in accordance with the critical size and specific gravity of the material available. As the random velocities which make up

the turbulent flow provide the means for carrying the particles into the interior of the fluid, a quantitative theory of turbulent flow also provides a method for defining distribution of material in suspension. The theory of turbulent flow, as developed by Reynolds, Prandtl, Von Kármán, and Taylor, has been applied to the distribution of suspended material by Schmidt (42) and O'Brien (36). More recently the principles of the problem have been briefly reviewed by Rouse.[6]

The cross currents or eddies which transfer momentum and result in an apparent shearing force, also provide the mechanism for distributing suspended material throughout the interior of a fluid. While the material in suspension falls relative to the fluid surrounding it, this effect is compensated by the transfer of material through eddy motion. Assuming that the average percentage of sediment decreases with distance above the bed, the upward velocities will carry with them more sediment than is returned by the downward velocities, and hence it is possible to attain a steady distribution of sediment.

The fundamental equation of convection due to turbulence is

$$\tau = \varepsilon \frac{d(\rho v)}{dy} \tag{22}$$

where the rate of transfer τ (the intensity of fluid shear) occasioned by the mixing process of the local characteristics of the fluid ρv (momentum per unit volume), or of the flow, must be proportional to the kinematic mixture coefficient ε and to the rate of decrease in the given direction.

The same basic relationship holds true for the vertical transfer of dust particles by atmospheric mixing, assuming a state of equilibrium to exist between the net convection due to particle weight; thus, denoting concentration of suspended matter by c and the terminal settling velocity by w,

$$cw = \varepsilon \frac{dc}{dy} \tag{23}$$

with y being measured in the vertical direction.

Integration of equation 23, assuming the mixing coefficient ε to remain constant with y, yields an expression for the concentration of a suspension at any level in terms of the concentration at some artibrary reference level a:

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[6] Rouse, Hunter. Laws of Transportation of Sediment by Streams: Suspended Load. Internatl. Union Geod. and Geophys. Round-table Discussion. Washington, Sept. 13, 1939. (Mimeographed.)

$$\frac{c}{c_a} = e^{-\frac{w(y-a)}{\varepsilon}} \tag{24}$$

Recent experiments by Hurst (17) and Rouse (40, pp. 550-554), involving the use of apparatus producing a constant intensity of mixing throughout a vertical column of water, has provided quantitative confirmation of the interrelationship of the several factors involved.

In actual streams ε varies not only with the distance above the bed but also from bank to bank. This is obvious if one regards ε as the product of the velocity of fluctuation of a length proportional to the size of the eddies. Since eddies grow in size with distance from the boundaries, the velocity of fluctuation is greatest in the boundary region, ε thereby attaining a maximum at some intermediate point.

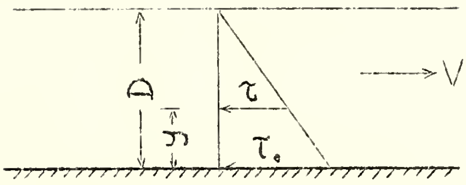
Assuming that the magnitude of the mixing coefficient is the same for the transfer of both momentum and suspended matter, this factor might be eliminated through the combination of equations 22 and 23. Integration for conditions of essentially constant density would then yield the expression,

$$\log \frac{c}{c_a} = w\rho \int_0^y \frac{\frac{dv}{dy}}{\tau} \cdot dy \tag{25}$$

where the distribution of shear and that of velocity gradient are still unknown.

In the case of steady, uniform flow, the magnitude of τ at any point may be determined by applying the principle that the total shear over the surface of a fluid element must be in equilibrium with a component of its weight acting in the direction of motion. Using published cross sections of several natural streams in which uniform flow was assumed, Leighly (29) obtained the distribution of shear and velocity gradient from plotted isovels. From these data the relative distribution of suspended matter was determined (fig. 8). A somewhat similar study was made by Christiansen (5) with particular regard to distribution curves in a vertical for different sediment sizes.

For flow in a very wide channel, the distribution of shear in the central region will approach that of truly uniform motion, namely, a linear function of the relative depth below the free surface:



$$\tau = \tau_0 \left(1 - \frac{y}{D} \right) \tag{26}$$

where D is the total depth and τ_0 the intensity of shear or tractive force at the bed. As such open-channel flow is the two-dimensional counterpart of uniform flow in circular pipes, it would seem reasonable to assume that the velocity distribution would be of logarithmic form. Von Kármán has suggested that the following expression be introduced into equation 25:

$$\frac{dv}{dy} = \frac{1}{k} \sqrt{\frac{\tau_0}{\rho}} \cdot \frac{1}{y} \tag{27}$$

which together with equation 26 upon integration yields

$$\frac{c}{c_a} = \left(\frac{D/y - 1}{D/a - 1} \right)^z \tag{28}$$

where z embodies the ratio of the fall velocity to the so-called "friction velocity" at the bed:

$$z = \frac{w}{k\sqrt{\tau_0/\rho}} = \frac{w}{kV\sqrt{f/8}} = \frac{w}{k\sqrt{gDS}} \tag{29}$$

In these expressions k is Von Kármán's universal constant ($k = 0.4$), V the mean velocity of flow, f the coefficient of bed resistance, g the acceleration of gravity, and S the slope of the energy gradient. It has recently been shown by Keulegan (21) that the foregoing assumption as to velocity distribution is valid for pipes and open channels alike.

Equation 28 has been recently experimentally verified at the Soil Conservation Service cooperative laboratory at Pasadena, Calif., through actual measurement of all the variables involved in the equation (see fig. 9).[7] Examination of this equation shows

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[7]Vanoni, V. A. Experiments on the Transportation of Sediments in Suspension. Investigation in progress (1940) at the Soil Conservation Service cooperative laboratory at the California Institute of Technology. (Unpublished.)

that it is possible to express the relative distribution of material of a given fall velocity in terms of easily measured flow parameters. The absolute amount of material in suspension is still indeterminate, for equation 28 presupposes knowledge of the sediment concentration at some reference level. It will be necessary to develop a similar function for conditions at the bed which will determine the absolute amounts of material carried into suspension.

An important step in this direction is suggested by Lane and Kalinske (28) who reason that (1) only those particles having a fall velocity lower than the instantaneous vertical fluctuation in bed velocity can be lifted off the bed, (2) the velocity fluctuations at the bed vary with time in accordance with the normal error law, which is fully characterized by the standard deviation from a mean zero; and (3) the root-mean square velocity fluctuation is directly proportional to the friction velocity. Application of the theory to field data shows that in general a prediction of suspended-material concentration from knowledge of the hydraulic characteristics and bottom composition of a river is quite possible within certain limits; however, carefully controlled laboratory experiments are needed to define the functional relationships.

Field Investigations

Estimates of the quantity of suspended matter carried by a river are necessary, in most instances, for the proper consideration of flood-control plans. Much of the material carried by a river may be deposited in proposed reservoirs and may eventually occupy a large part of the capacity of the reservoirs so that their effectiveness for water storage, flood control, or river regulation will be greatly decreased. The quantity of suspended matter carried by a river usually varies considerably from year to year. The annual suspended load in small flashy streams is carried within a small number of days out of the year and silt content of the water may vary considerably within a short-time period on the rising or falling stage of the stream (30). For instance, figure 7 shows discharge plotted against concentration of suspended matter for a particular flood on Coon Creek at Coon Valley, Wis.[8] The importance of careful and frequent sampling[9] during floods is obvious from examination of this plotting; that is, if only a single sample were taken to estimate the daily stream load, a sample taken at 5:40 a.m. applied to the mean daily discharge would give a load of

[8] U. S. Geol. Survey Water Resources Branch. Determinations of Loads of Suspended Matter in Small Streams. Quality of Water Division, May 10, 1939. (Mimeographed.)

[9] A complete description of various types of suspended-load sampling equipment is given by E. W. Lane (26).

64 tons per day and a sample taken at 8:00 a.m. would give a load of 22,700 tons per day, but by planimetering figure 7, which is plotted from 21 samples, the true load is found to be close to 7,660 tons per day. Thus, it is seen that the reliability of the results is dependent largely on obtaining sufficient samples at the right time.

As has been mentioned before there is never any fixed or even an approximately constant relation between discharge and concentration of suspended load. The reason for this lack of relationship is that the suspended load consists mainly of very fine material whose occurrence in a stream depends on factors other than discharge. In other words, it is impossible at present to predict even approximately the quantities of material that will be washed into a stream within a given period of time as the result of erosion of a drainage area.

The possibility of estimating, with any degree of accuracy, the total suspended-silt content in any stream at any stage of a given rise, even with the records of many rises available, is extremely questionable. In many cases, however, it is possible to determine approximately the average amounts of the various particle sizes usually carried by a stream by making mechanical analyses of a large number of samples of suspended matter. For instance Vetter (56) found, on the Colorado River, a definite relation between load and discharge for particles larger than 0.03 to 0.01 millimeter. For material smaller than this size range, however, he found no apparent relationship. Lane (27) has attempted to explain this lack of relationship as being due to the fact that the relationship between fall velocity and particle size changes abruptly at this position in the size range.

In most of the investigations initiated to determine potential damage to reservoirs, etc., no velocity observations were made and only determinations of total suspended load without separation into the various particle sizes were made. Consequently most of the suspended-load data are valueless in giving information with which the recent theories of sediment transportation can be checked.

SUMMARY

The primary purpose of investigations on the transportation of debris is to obtain basic data (usually by field measurements) relating to the magnitude and nature of the sedimentary load of a particular river, and also to determine the fundamental laws (by laboratory studies and field verification) governing the sedimentary characteristics of any river.

The amount of silt transported in suspension by a stream varies between wide limits and depends upon so many factors that a relation between discharge and quantity of suspended sediment cannot be accurately determined. Continuous sampling is the only method by which an accurate estimate of the suspended load of a stream can be determined.

On the other hand, a practical method has not been devised as yet for directly measuring the quantity of sediment transported by a large river along its bed. Several types of portable traps have been suggested at various times, but none, except possibly the Arnhem trap of Schaank, appears to be satisfactory for making quantitative determinations of the bed load of rivers. For determining the amount of sediment transported over a long-time period in a particular stream, measurements have been made on the rate of sediment deposited in reservoirs formed by dams constructed across the stream. This method produces certain difficulties because, although daily observations may be made of the silt discharge into and out of the reservoir, there is the uncertainty of the weight of the material per unit volume and, therefore, uncertainty of the space occupied by the suspended and bed loads respectively (53). Although the total sedimentary load, consisting of the suspended load (estimated by sampling) and bed load (estimated from portable traps or by application of a rational formula), is the maximum amount of material that may enter a potential reservoir and be deposited, some of the finer material will certainly pass through almost any reservoir either by direct flow or as a density current. Another matter of interest is that the sedimentary load of a stream and its tributaries provides an index to the seriousness of erosion in the various parts of the drainage basin. The quantity of material in movement, however, is probably less than the total amount eroded because much of the material scoured from the highlands comes to rest, through deposition, in the low lands without being carried into the main river.

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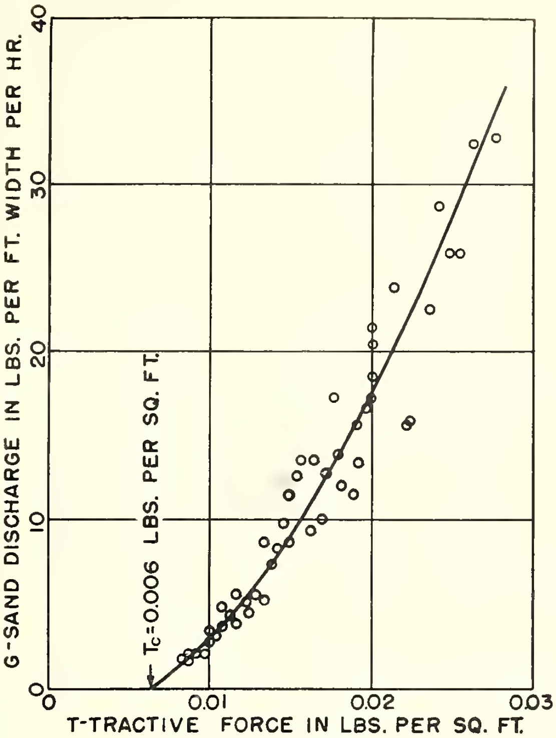


FIGURE 1-LINEAR PLOT

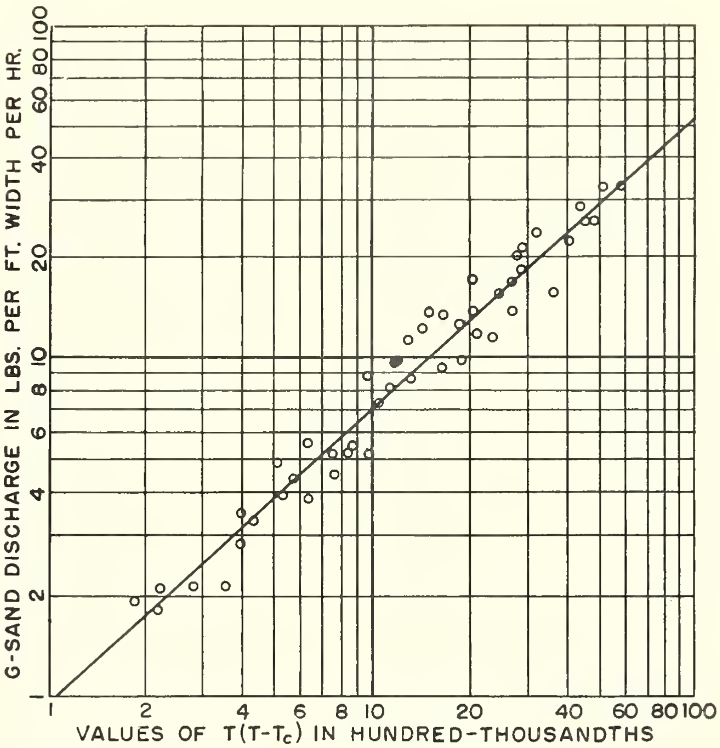


FIGURE 2-DU BOY PLOT

TRANSPORTATION OF BED LOAD (U.S. WATERWAYS EXPERIMENT STATION SAND NO. 1)

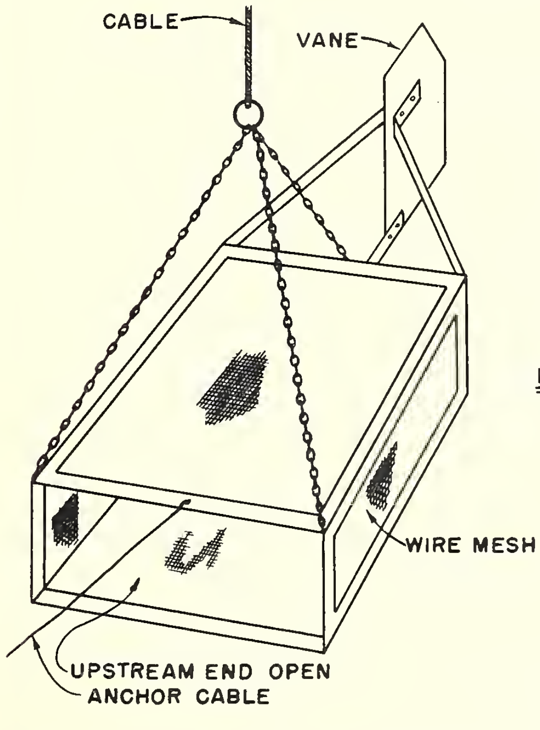


FIGURE 3-RHINE BED-LOAD TRAP
(EINSTEIN)

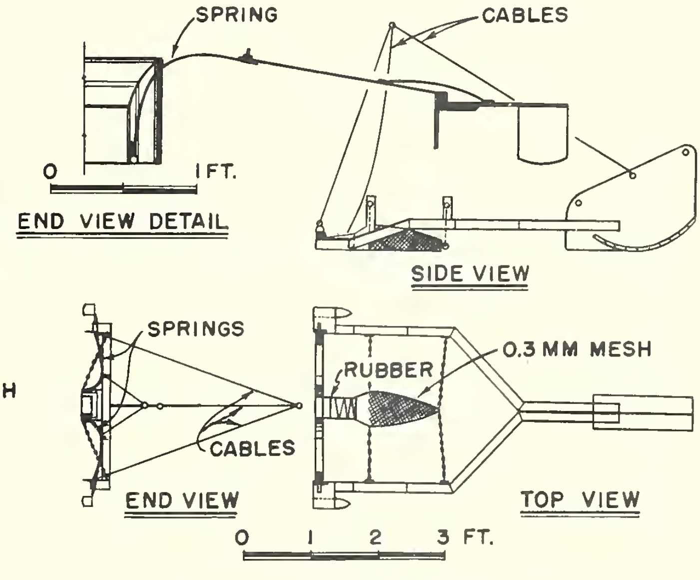


FIGURE 4-ARNHEM BED-LOAD TRAP
(SCHAANK)

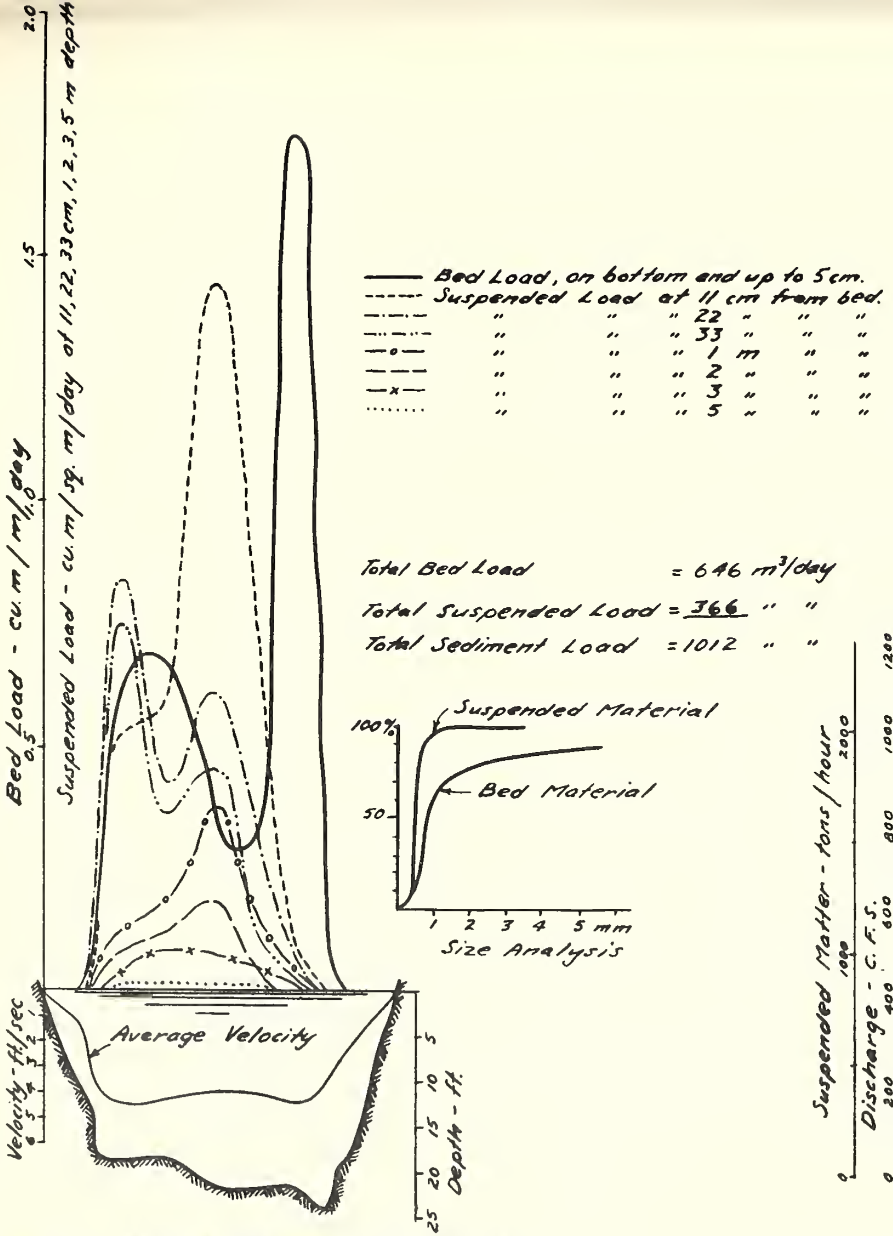


FIG. 5 — SAND TRANSPORTATION IN THE LOWER RHINE (SCHAANK).

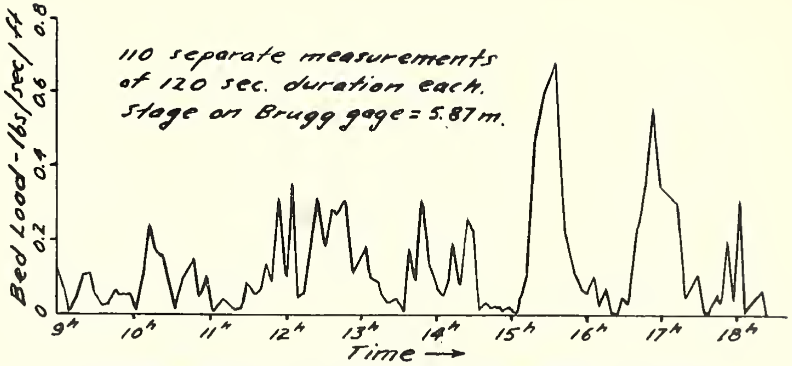


FIG. 6 - VARIATION OF BED-LOAD MOVEMENT IN THE RHINE AT BRUGG (MEYER-PETER).

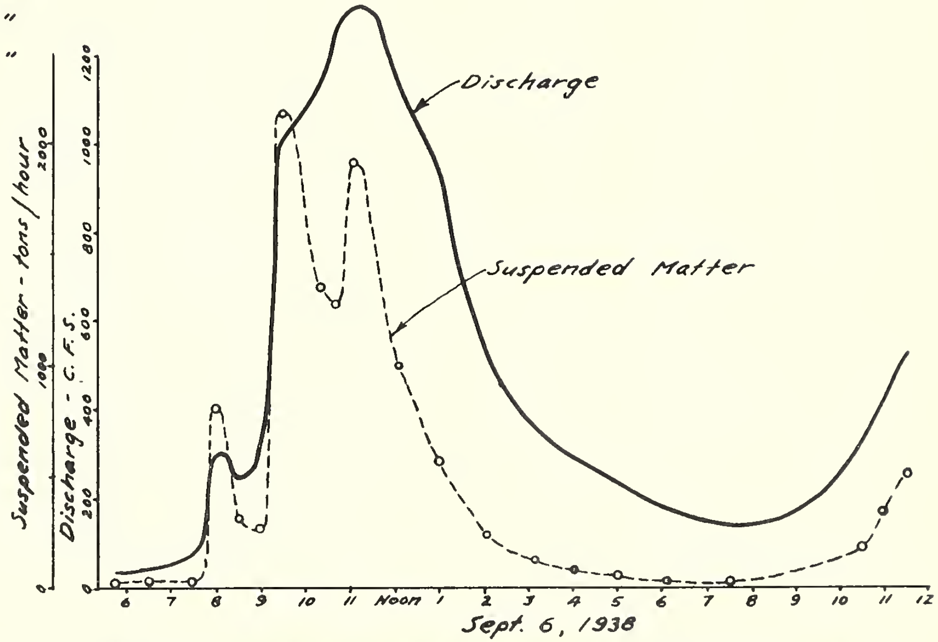
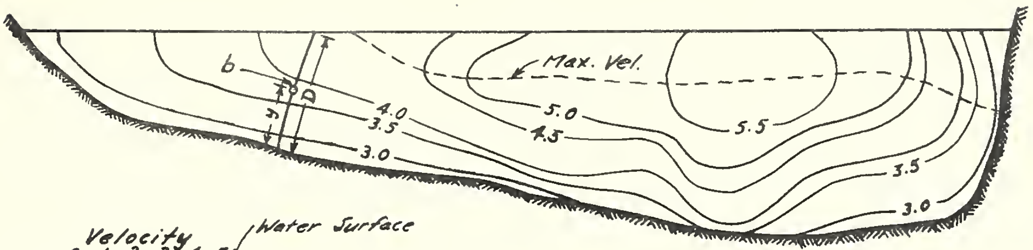
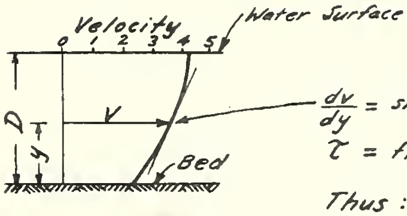


FIG. 7 - DISCHARGE AND SUSPENDED MATTER, COON CREEK AT COON VALLEY, WIS. (COLLINS).





$\frac{dv}{dy}$ = slope of tangent to velocity curve at point "b".

τ = fluid shear at point b = $(D-y)\gamma S$

Thus: $\quad \varepsilon = -\dfrac{(D-y)\gamma S}{\left(\frac{dv}{dy}\right)_b}$

∴ The sediment concentration at "b" in terms of a known concentration at some arbitrary reference level "a" is: $c = c_a e^{-\frac{w(y-a)}{\varepsilon}}$

FIG.8 - LEIGHLY METHOD OF CALCULATING THE DISTRIBUTION OF SUSPENDED MATTER IN OPEN CHANNEL.

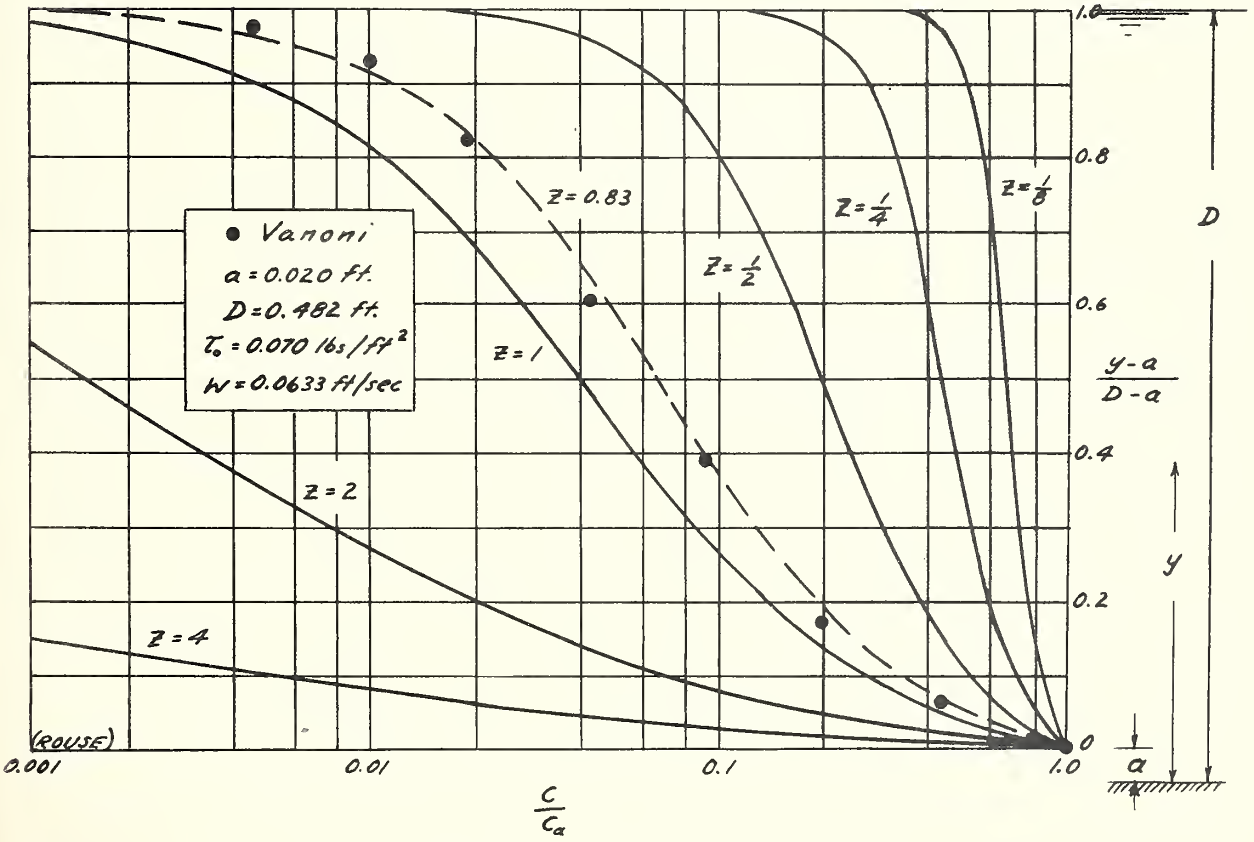


FIG.9 - RELATIVE DISTRIBUTION OF SUSPENDED LOAD IN AN OPEN CHANNEL.